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DEVELOPMENT OF METHODS FOR ADJUSTING THE USSR ASTRONOMIC-GEODETIC NETWORK

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FOREWORD

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DEVELOPMENT OF METHODS FOR ADJUSTING THE USSR ASTRONOMIC-GEODETIC NETWORK

The simultaneous adjustment of 87 polygons of the main triangulation of the USSR, projected on Krasovskiy's reference ellipsoid, made possible the establishment of the "1942 System of Coordinates" in place of the "1932 System of Coordinates" and the "Svobodny System". In the near future it is expected that all field work in first order triangulation will be completed. This brings up the question of the best method of adjusting the large system of polygons of the astronomic-geodetic net. In connection with this it is proposed to make a full preparatory investigation of specific characteristics of the first order triangulation of the USSR, methods of adjusting it and modern views of the latter.

1. Some Characteristics of the Astronomic-Geodetic Network of the USSR

The astronomic-geodetic net of the Soviet Union must insure the accelerated development of the geodetic control base over a very wide area (over 22 million km²) and serve the scientific aims of higher geodesy. With a view toward attaining these ends the establishment of the astronomic-geodetic net is accomplished in the form of triangulation arcs oriented primarily along meridians and parallels. As the result of the intersection of the arcs the astronomic-geodetic net was created in the form of systems of polygons. (Polygons of intersecting chains or arcs) The scheme and program of establishing this net were developed by F. N. Krasovskiy Ref (1) and later revised by commissions of the GUGK (Glavoye Upravelniye Geodezii i Kartografii - Chief Administration of Geodesy and Cartography) under the direction of S. G. Sudakov. The revisions were in relation to the dimensions of the polygons and the accuracy of angular measurements. Ref (2)

The main triangulation must be established with a very high accuracy in accordance with modern methods and means of measurement. Therefore, increasing the accuracy of angular measurements and shortening the sides of triangles which, in turn, shortens the sides of the

polygons, contrary to the scheme and program of Krasovskiy, are measures to improve the construction of the astronomic-geodetic net of the USSR.

The carefully planned and scientifically founded system of establishing the huge astronomic-geodetic net of the USSR acquires an especially important significance. Without using this net, deductions of new data on the figure or dimensions of the earth can not be conclusive.

It is especially important to note that the great amount of data on the results of angular measurements and on azimuthal and base line errors of closure in sections (Note: A section is a chain between two bases.), projected on the reference ellipsoid, give a basis for conclusions in the study of systematic errors which accompany angular measurements. An analysis of the character of free terms of zaimuthal and base equations led to the disclosure of systematic errors in the measurement of angles and to the noting of means of lessening their effect. Ref (3) Ref (4).

In small trigonometric nets systematic errors of angular measurements are overshadowed by the effect of accidental errors, and, conversely, in the astronomic-geodetic net of the USSR in the influence of systematic errors of angular measurements is very obvious. In the latter case, reference is made to those systematic errors of angles which can be attributed to the overall geographic distribution of atmospheric density and which systematically distort the oreintation and scale of the triangulation.

Atmospheric density increases from the equator to the poles. The angular distortion from lateral refraction which arizes from this condition must lead to a systematic distortion of the free terms of the azimuthal and base condition equations. If the initial azimuth and base are transmitted from Pulkovo eastward in arcs (of triangulation) along parallels, and southward along meridians, then, under the influence of refractional distortions of angles, we theoretically must get

$$W_{a(p)} > 0, \quad W_{b(p)} \geq 0$$
$$W_{a(m)} > 0, \quad W_{b(m)} < 0$$

Where $W_{a(p)}, W_{a(m)}$ - are the azimuthal free terms for sections corresponding to parallels (p) and meridains (m) and $W_{b(p)}$ and $W_{b(m)}$ are the base free members.

As we showed (Ref 3), the inequalities (1) find practical verification in long arcs over the entire adjusted part of the net (311 sections). Namely for arcs along meridians and parallels

$$\sum W_a = \sum \sum (\delta c_i) + (\delta \omega_0 - \delta \alpha_m) > 0,$$

for arcs along parallels

$$\sum W_b = \sum \sum [\Delta_A (\delta A_i) - \Delta_B \delta B_i)] + (\delta \rho \cdot \delta b_m).$$

and for meridinal arcs

$$\sum_i^m W_b < 0$$

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In expressions (2) and (3), $(_b A_i)$, $(_\ B_i)$, $(_\ C_i)$ are the refractional distortions in the tieing and intermediate angles of the sections of triangulation; $_\delta$ a_m, $_\ b_o$, $_\ b_m$ are the errors in the initial and last Laplace azimuth and base in the section; m is the number of sections in the arc and $\triangle_A$, $\triangle B$ are the increments per second of the log sines of the angles.

From (2) and (3) it is seen that the errors in angles A_i, B_i, C_i affect mainly $\succeq W_a$ and $\preceq W_b$. The errors in the azimuths and lengths of the starting sides (Note: starting or initial side is the expanded base line.) in intermediate sections are excluded and there remain only the difference of those errors $(_\ a_o - _\ a_m)$ and $(_\ b_o - _\ b_m)$ for the initial and final sides of the arc. In other words, in the build up of the systematic distortions $\sum W_a$ and $\succeq W_b$, the errors in the azimuths and lengths of the initial sides have almost no significance. Some idea of the magnitude of the differences $(_\ a_o - _\ a_m)$ and $(_\ b_o - _\ b_m)$ is possible partly from the results of substtituting in along arc, the first and last section with other sections which join the same junctions. Experience has shown that such substitutions do not make essential changes in $\preceq W_a$ and $\succeq W_b$.

As an illustration of the accumulation of systematic distortions in $\succeq W_a$ and $\succeq W_b$ let us cite some examples on the longest arcs.

1) Arc along the parellel Orsha-Ferma, $\varphi_m = 55°0$, distance 7400 km, 37 sections, $\succeq W_a = +13".7$, $\succeq Wb = +744 \times 10^{-7}$ (in log units)

2) Arc along the parallel Prokop'evsk-Khabarovsk   m = 52.°7, distance 3800 km, 17 sections, $w_a = +20".3$, $W_b = +220 \times 10^{-7}$.

3) Arc along the meridian Arkhangel'sk-Zugdidi, $\lambda_m = 40°.7$, distance 2500 km, 12 sections, $\preceq W_a = +12".3 \succeq Wb = 220 \times 10^{-7}$.

(4 Arc along the meridian Novosibirsk-Narym, $\wedge m = 80.°4$, distance 1700 km, 9 sections, $\succeq W_a = +18".5$, $\preceq W_b = -74 \times 10^{-7}$.

5) For 20 arcs along the parallels of distance 32,500 km we have: $\succeq W_a = +58"$, $\preceq W_b = +3083 \times 10^{-7}$ in log units.

6) For 24 meridianal arcs of distance 21,700 km we have: $\succeq W_a = +70".3$, $\succeq W_b = -1025 \times 11^{-7}$.

The many forms of physical and geographic conditions over the territory of the USSR give rise to large regional fields of lateral refraction which are not subject to the general geographic distribution of atmospheric density with a gradient index of refraction in an approximate northeast direction. (ref 4). For certain arcs it can even lead to a violation of the inequalities (2), (3), and (4). However, the overall pattern of systematic distortions in azimuthal and base errors of closure is in keeping with these inequalities.

From data on 87 polygons of the astronomic-geodetic net of the USSR, the mean systematic distortion (error) of free members of azimuthal and base conditional equations in a section is: $_\ W_a = +0."5$, $_\ W_{b(p)} = +19 \times 10^{-7}$ and $_\ W_{b(m)} = -9 \times 10^{-7}$. In this fashion, in large "polygonal" triangulation, ocomputed by the method of projection, and therefore free from distortions of geometric origin,

systematic distortions in its orientation and scale disclose themselves rather clearly. The most essential characteristic of large astronomic-geodetic nets is the preponderance of accumulation of systematic distortions in orientation and scale over the analogical accumulation of accidental distortions. In developing methods of adjustment, this characteristic must be kept in mind. Besides this it must be mentioned that in adjusting large astronomic-geodetic nets there arises the problem of simultaneous solution of systems from a very large number of equations. In this case the solution can not lead to the violation of the properties of the first system. The roots found must correspond to it.

2. General Considerations in the Adjustment of a Large Polygonal Astronomic-Geodetic Net

The primary conditions in establishing a high quality astronomic-geodetic net are: high precision instruments, their careful use in accomplishing measurements and the character of the construction. In such a case we must use a method of computation in which the very accurately measured field values would not obtain inadmissable additional distortions as a result of the adjustment. The corrections obtained in the adjustment must in general correspond to the errors of measurement. The adjusted elements must be as a near as possible to the most probable. With the present state of adjustment computations, these requirements can be fulfilled by means of simultaneous solution of equations of the net under the condition (pv^2) = min. with the utmost rigidity. It must be kept in mind that in triangulation there is no such thing as absolute correspondence between measured values and their assigned weights and that there are no absolutely independent values. The character and purpose of measured values in triangulation are essentially differnet (varied). We have here a very large number of independently measured horizontal directions which allow us to compute from a single datum point and a single side measured in length and in azimuth, with sufficient check, the coordinates of all the points in the net. However, besides the horizontal directions, we provide in substantially less numbers, Laplace azimuths and lengths of initial sides of base nets. For instance, in the adjusted part of the astronomic-geodetic net (87 polygons), there were measured some 14,000 horizontal angles, 220 Laplace azimuths and the same number of bases. The purpose of these azimuths and bases was to maintain the orientation and scale of the triangulation by sections and in the entire net as a whole. These values, unlike horizontal directions, should lessen the latitudinal and longitudinal displacements, especially displacements of a systematic character which are strongly apparent in an astronomic-geodetic net. Consequently, azimuths and lengths of initial sides paly an essentially different role than horizontal directions. They are the "standards" of orientation and scale of the net. The triangulation net is based on them, hence, a part of the systematic errors of the angles can not be laid on them.

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However, we must note that azimuths and bases, like measured horizontal directions, are not free of errors. There arises the question as to how to exclude horizontal directions from the effects of errors in azimuths and bases.

We can propose several variants for solving this problem. Assuming as the first variant the inclusion of azimuths and bases as values which are adjusted with a consideration of their wieghts. Formally, this is the most rigid variant. We must only keep in mind that the role of weights in the distribution of free members of zaimuthal and base equations can be violated by the configuration of the net and not infrequently the essential non correspondence of the mathematical value of the weight with the physical conditions of the measurements.

The second variant is the establishment of such frequency of distribution of azimuths and bases in triangulation, that the influence of those values in the free members of azimuthal and base equations would be negligible; that is, it would conform to the criteria of insignificance in comparison to the effects of errors in azimuths and bases together with the influence of errors in horizontal directions. Consequently, this would allow the acceptance of azimuths and bases as concrete values, that is, not adjustable.

The third variant originates in the location of azimuths and bases every 200 km as practiced in the USSR. It takes into consideration that with such frequency of distribution and unequal accuracy of these elements, their errors are not insignificant in all sections of triangulation. Nevertheless, in this variant, azimuths and bases are accepted as true, since the accumulation of azimuthal and base errors of closure W_a and W_b in a large astronomic-geodetic net maintain a systematic character and the adjust ment of azimuths and bases leads to a proration to them of some portion of these closures conditioned by angular systematic errors. It follows that we can not allow the distortion of orientation and scale of triangulation on account of systematic errors of angles.

Let us investigate which one of these variants is the best founded and most acceptable in practice.

The practical use of the first variant requires the accurate settting up of weights that are completely real in the physical sense for azimuths and lengths of initial lines. As relates to the length of sides, the solution of this problem is possible in each individual case which is sifficiently controlled starting with the precision obtained in measuring the given base and the errors found while computing the lengths of the starting line in adjusting the base net. However, modern instruments and methods for base measurement insure high accuracy of determining the lengths of bases. From the adjustment of a base net, the initial side is computed with a relative accidental error usually, less than 1: 350,000. The effect of such an error on the magnitude of the free member of a base adjust ment in comparison to the effect of errors of angular measurement, is insignificant. This principle was adhered to in the adjustment of triangulation of the USSR

as relates to errors of initial lines in the second variant. Isolated proposals to include the lengths of initial sides as adjustable values have not encountered serious acceptance at the present time. Therefore, we will further investigate the use of avariants for Laplace azimuths only. It must be kept in mind that we do not have data for fully founded establishment of weights for these azimuths. As we showed in our work (ref 5) neither the value of the mean square error M_a of 18 positions (readings) of an observed astronomic azimuth nor the value W_a of the free member of a Laplace equation from reciprocally observed azimuths are criteria for verified determination of the weight of a Laplace azimuth, the latter having been adjusted on the basis of results of astronomic observations on two stations of a base net. Besides this, in case of a small value for W_a, the adjusted value of the azimuth may be essentially less accurate than when W_a is considerable.

Even with suitable data from azimuthal observations for deducing real physical errors and establishing from them the weights, there is the possibility because of the shape of the net, of a violation in the distribution of azimuthal closures W_{ik} in the sections. The distribution may sometimes not correspond to the established wieghts.

As an example, let us look at some data from the astronomic-geodetic net of the USSR. In adjusting 87 polygons the same weight was used for all Laplace azimuths, the weight being unity. All azimuths had been determined from forward and back observations. An azimuth observed in a single direction only was assigned weight 0.5. Then there was formed a system of 311 conditional azimuthal equations (Ref 6)

$$\Sigma \Delta_i + u_i - u_k + W_{i,k} = 0$$

where $_i$ is the correction given to all measured directions ik in the section (the reciprocal of the weight $1/p \Sigma \Delta_i$ of this correction is determined in the adjustment of the section and averages about 4; u_i and u_k are the corrections to the Laplace azimuth depending on the errors $\delta\alpha, \delta\lambda$ of the azimuth and longitude ($u_i = \delta\alpha_x - \delta\lambda_r \sin\theta$); $W_{i,k}$ is the free member of the equation.

In assigning the same weight, unity, to all double Laplace azimuths, A; it is difficult to visualize any real, physical meaning. The variations in magnitudes of the free members W_A, mentioned above, indicate (Ref 5) the unequal accuracy of the azimuths A_i. Besides this, an analysis of the magnitudes of the corrections $\Sigma \Delta_i$, u_i, u_k reveals, in one third of the cases, a non-correspondence of these magnitudes with assigned weights (here we have a distortion of the shape of the net.) Even if the weights u_i and u_k are greater than the weights $\Sigma \Delta_i$, nevertheless the magnitude of the corrections u_i and u_k often exceed significantly the corrections $\Sigma \Delta_i$. This condition is illustrated by a few examples in Table 1. $W_{ik} = \Sigma \Delta_i - u_i + u_k$

TABLE 1

Section (Chain)	W_{ik}	$\sum \Delta i$	u_i	u_k	$\frac{1}{F_2}\Delta i$	$\frac{1}{u_i}$	$\frac{1}{u_k}$
Petrozavodsk-Lodeyn. Pole	1.72	0."37	0."13	1."22	1.4	1.0	1.0
Arkhangel'sk-Plesetskaya							
Pereyaslavl'-Kremenchug							
Buzuluk-Zirgany							
Zima-Irkutsk							
Uglich-Kostroma							
Neya-Vologda							
Birobidzhan-Khabarovsk							

Thus, the distribution of azimuthal closures W_{ik} into parts corresponding to section errors $\sum \Delta i$ of angles and errors u_i and u_k of Laplace azimuths, is often in contradiction to the weights.

From these rules and facts it follows that the first variant of using azimuths as magnitudes which are unconditionally adjusted, is not without fault. In the first (variant) the accepted weights of azimuths willnot be free from errors which creep in from the actual accuracy of adjusted Laplace azimuths. In the second, the degree of correspondence between the determined and the adjusted corrections and their weights depends upon the shape of the net so that distribution of closures into parts by weights can be contradictory.

The second variant should be the best. Its use requires that the errors in lengths azimuths of initial lines satisfy a criteria that they be insignificant as compared to those errors of base and azimuthal closures in sections of triangulation. If the mean square error m_F of a function is composed of the mean square errors m_1 and m_2 of two (or more) arguments, then the criteria of insignificance of one of these errors will be (ref 7):

where we assume

$$m_1 \leqslant \tfrac{1}{3} m_F$$

$$m_F = \pm \sqrt{m_1^2 + m_2^2}$$

Depending on the methods and means of determining the lengths and azimuths of initial lines, we can find the mean square errors m_s and m_a and establish such frequency of locating initial lines that the values m_s and m_A would be one third of the mean square value of the free terms of the base $m_W(b)$ and azimuthal $m_W(a)$ conditional equations.

Let us investigate what are the mean relationships $|m_{1_a}\sqrt{2}| : |m_v(b)|$ and $|m_A \sqrt{5}| : |m_{(w \omega)}|$ from data of the 87 adjusted polygons of the

astro-geodetic net of the USSR. For this purpose we will avail our-
selves of the following expressions of the mean square value of free
terms of base and azimuthal equations in a section of triangulation:
(Ref 8)

$$m^2_{\omega(a)} = 2m^2_A + \frac{k+3}{2} m''^2,$$

$$m^2_{\omega(b)} = 2m^2_{lgs} + m''^2 \sum^k (\Delta^2_A + \Delta^2_B),$$

where m_A is the mean square error of the Laplace azimuth, $m_{log\ s}$ is the
mean square error of the log of the length of the initial sides, m is
the mean square error of measuring first order triangulation angles, k
is the number of triangles in a section, Δ_A Δ_A and $_B$ are the in-
crements per second of the log sines of the distance angles A and B.

The mean number of triangles k in a section of first order tri-
angulation in the USSR (from 310 sections) is 15 and m'' = ± 0."7. For
the mean square values of the Free terms we have (Ref 4 and (Ref 9):

$$m_{W(a)} = 2."9 \; ; \quad m_{W(b)} = 81 \times 10^{-7}$$

The relative error Δ s/s of the initial side does not, as a
rule, exceed 1: 350,000, therefore $m_{lg\ s} = \Delta s$ u = 12.3 X 10^{-7} (u
0.43429 ....). As far as the errors m_A of the Laplace azimuth which
have been adjusted from reciprocal observations are concerned, they
can be found from the equation (7). From this expression, and using
the numerical values given here we obtain:

$$2 m^2_A = 4, m^2_{\omega(A)} = 8.412 \frac{k+3}{2} m''^2 = 4.41, m_A \sqrt{2} = 2,$$

then

$$|m_A \sqrt{2}| : |m_{\omega(a)}| = 2.0 : 2.9 = 0.7 > 1/3$$

Further we have $m_{ks} \sqrt{2} = 12.3 \times 1.4 = 17.3$ in units of the 7th place
and

$$|m_{lgs} \sqrt{2}| : |m_{\omega(b)}| = 17:81 == 0.22 < 1/3$$

From these calculations it is seen that the value m_A $\sqrt{2}$ does
not answer to the criteria of insignificance. The effect of this value
is a magnification of the mean square value of the free term of the az-
imuthal equation from ± 2".1 to ± 2".9 while the effect of $m_{lg\ s}$ $\sqrt{2}$
is to increase the value of the free term of the base equation overall
from ± 79 X 10^{-7} to ± 81 X 10^{-7}, that is, by 1: 2X10 ^{6}.

Thus, the accuracy of Laplace azimuths does not correspond to
the accuracy of initial lines and with the accepted frequency of the
latter, that is, every 200-250 km, it does not satisfy the criteria
of insignificance. The second variant, which is the most accepted,
does not suit the astro-geodetic net of the USSR. It is necessary that
$m_A \sqrt{2} < ± 1."0$ or that $m_A = 0."7$ whereas actually $m_A = ± 1".4$. This
follows, however, only under the condition that the value m_A is determined

less accurately than the value $m_{lg\ s}$, since m_A contains the influence of systematic angular errors which are not considered in $\underline{k + 3\ m''^2}$.
$$2$$

There remains the use of the third variant, in accordance with which is allowed a mean square error for the entire sum of the 45 angles of a section of $\pm 2''$.

The results of special studies, verifying the foundation and superiority of using the third variant, are given below. It is clear, however, that in the simultaneous solution fo all equations arising in the given astro-geodetic net, under the condition $(pv^2) = \min$, we must give special consideration in the net to the lengths and Laplace azimuths of the initial lines. We can not lay upon them the systematic errors of base and azimuthal closures which accumulate in a large net. It is more feasible to add to the total of 45 angles of a section, the additional accidental distortions of azimuths of about $\pm 2''$, than to carry over to the azimuths the effects of systematic angular errors.

Let us mention one more factor which is important in a large astro-geodetic net. The rigid adjustment of such a net under the condition where $(pv^2) = \min$ requires the simultaneous solution of systems from a very large number of normal equations and is a rather difficult assignment. Therefore, in the previous century as well as in the first half of this century, for well known reasons, large nets had to be adjusted by stages, in parts and non-rigidly. Instead of making corrections to directly measured directions, they were made to mutually dependent functional relationships. A method for adjusting such relationships were developed by the German geodesist F. Helmert.

Let us pause for a general study of this method which was further developed in the works of Krasovskiy for better adaptability to the adjustment of polygonal astro-geodetic nets.

3. General Facts on Helmert's Method and the Revisions Made Thereto by Krasovskiy.

In the 80's of the last century, Helmert showed a way of adjusting an astronomic-geodetic net. It consisted of three stages:
1) independent adjustment of each section of first order triangulation,
2) simultaneous adjustment of polygons formed by the diagonals of the adjusted sections,
3) placement of the triangles of the section between the adjusted coordinates of the apexes of the polygons.

In adjusting each section, there were set up conditional equations for figures, poles and bases. There was no azimuthal equation in this stage.

Then the functional relationships, which were computed from the adjusted elements of the section, were used to form polygons. They were, the lengths s_{jk} of the geodetic lines (Figure 1) and the angles

Figure 1

$$B_{JK} = A_{JK} - A_{Jm}, \quad B_{KJ} = A_{KJ} - A_{Kn},$$

where A_{JK}, A_{KJ} are the azimuths of the geodetic line of the section JK and A_{Jm}, A_{Kn} are the computed geodetic azimuths of the initial lines I_m and K_n (the Laplace azimuths of these lines are indicated as $A_{(Jm)}$, $A_{(Kn)}$).

In the simultaneous adjustment of polygons there occurred the following conditional equations: latitudinal, longitudinal, azimuthal (Laplace equation) and the condition of the sum of the angles of the polygon. The latter equation occurs when all the apexes of the polygon do not coincide with Laplace stations (such cases do not exist in the astro-geodetic net of the USSR).

Conditional equations of polygons in Helmert's method have the following characteristics.

Besides the corrections $\int s$ to the geodetic lines, v to the angles B, $\int \alpha$ and $\int \lambda$ to the astronomic azimuths and longitudes, the equations still contain the unknowns Δa, $\angle_{\alpha}$, which are the corrections to the major semi-axis a_0 and the flattening a_0 of the assumed ellipsoid and orientation elements (the deflections of the verticals ξ_1, λ_1) in the triangulation datum point. The derived system of conditional equations including the unknowns $\Delta \alpha, \Delta \alpha, \xi_1, \Lambda_1$ (more accurately, $\Delta \alpha, \Delta \alpha (\xi - \delta \Phi_1), \eta, \sec \Phi_1$) is solved according to Helmert under the condition:

$$[P_s \delta s^2] + [P_v v^2] + [P_\alpha \delta \alpha^2] + [P_\lambda \delta \lambda^2] = MIN.$$

The terms of the polygonal equations into which enter the unknowns $\Delta \alpha, \Delta \alpha, \xi_1, \eta_1$ are included in the free terms W_i by Helmert. Therefore, in solving the normal equations, the numerical values of the correlates k_i are not yet obtained. However, the correlates, and as a result the corrections $\delta s, v, \delta \alpha, \delta \lambda$ can be presented in the form of linear functions of the above mentioned unknowns.

It is not necessary here to explain the rather exhaustive method of computation which included several stages and is intended in the final analysis to arrive at the above mentioned polygonal equations. Each stage of the computations is set forth in detail by Krasovskiy (Ref 10). Let us note only that for obtaining the correlates of the corrections $\delta s, v, \delta \alpha$ and $\delta \lambda$ which are expressed in the linear form as $\Delta \alpha, \Delta \alpha, \xi_1, \eta_1$, we can substitute one of the computation stages we set up, the expressions for declinations of the plumb lines ξ_i and η_i for all the astronomic points in the net. Thus, the declinations of the plumb lines on all astronomic stations will be expressed as functions the (named) unknowns

$$\xi_i = f_i(\Delta \alpha, \Delta \alpha, \xi_1, \eta_1)$$
$$\eta_i = \gamma_i(\Delta \alpha, \Delta \alpha, \xi_1, \eta_1)$$
$$i = 1, 2, 3, \dots$$

It is obvious that the functions (10) are error equations. Applying, as used in the derivation of elements for the earth's ellipsoid from degree measurements, the requirement $\sum \varepsilon^2 + \sum \cap \lambda^2$ = min., we can switch to the system of normal equations. From the solution of these equations we obtain the unknowns Δu, Δa, ε, Λ. With these unknowns and the free terms W of the conditional equations, we compute the correlates k_i, then the corrections $\iota s, V_i, \delta \omega, \delta \lambda$.

In this fashion, the adjustment of the astro-geodetic net by Helmert's method, solves tow problems at the same time: 1) derivation of the reference ellipsoid and 2) adjustment of the polygonal net.

In 1931, in the Works of the State Institute of Geodesy and Cartography, F. N. Krasovskiy published a method for adjusting the state first order triangulation (Ref 11). This method differed from Helmert's in the separate solution of the two problems mentioned. The derivation of the reference ellipsoid from degree measurements was seperated from the adjustment of the Astro-geodetic net. In this way, the exceptionally voluminous computations of Helmert's method were eliminated.

Krasovskiy's method, summarized in the "Works" (Ref 11), was used for the simultaneous adjustment on the Bessel ellipsoid of eight polygons of first order triangulation in the european USSR. This adjustment, as we know, gave rise to the "1932 System of Corrdinates".

The experience gained the adjustment of eight polygons was utilized by Krasovskiy in the later development of his method of adjustment. The latter was founded on his own developed method of projecting first order triangulation on a reference ellipsoid.

An adjustment by Krasovskiy's method of 1931 consisted of the following stages: 1) the adjustment of each section by directions for conditions of figure, azimuth, poles, and base * (Footnote: In Helmert's method, there was no azimuthal equation in a section adjustment).; 2) the adjustment of the system of polygons under conditions (9) for the equations of latitude, longitude and azimuth which appear therein and then the computation of the coordinates of the apexes; 3) the adjustment of the junction figures in order to isolate the sections from one another; 4) the placement of the chains of triangles between the accepted (adjusted) sides of the junction figures (Figure 2)

The separation of the adjustment of the net from the drivation of a reference ellipsoid eliminates from the polygonal equations the unknowns Δa, $\Delta \omega$, ε, Λ . For instance, in a quadrilateral polygon (Figure 2) the conditions of latitudes, longitudes and azimuths (Laplace equation), according to Krasovskiy, taking the following form

$$(\rho_1{}^{23}\rho_3{}'^2 + \rho_1{}^{7\cdot23}\Gamma_3{}'^2)\delta_{S_{/2}} + \rho_3{}^{23}\delta_{S_{23}} - (\rho_1{}^{43}\rho_3{}'^4 + \rho_4{}^{43}\Gamma_3{}'^4)$$

$$\delta_{S_{14}} - - \rho_3{}^{43}\delta_{S_{43}} - \rho_4{}^{23}(\upsilon_{31} - \upsilon_{23})(\rho_1{}^{43}\rho_4{}'^4 + \rho_4{}^{43}\Gamma_4{}'^4)$$

$$(\upsilon_{14} - \upsilon_{12}) - - \rho_4{}^{43}(\upsilon_{43} - \upsilon_{41}) + W_\beta = 0$$

$$(q_1{}^{20}p_3{}^{12} + q_4{}^{23}r_3{}^{12} + q_3{}^{12})\delta s_{12} + q_3{}^{23}\delta s_{23} - (q_1{}^{43}$$
$$P_3{}^{14} + q_4{}^{43}r_3{}^{14} + q_3{}^{14})\delta s_{14} - - q_3{}^{43}\delta s_{43} - q_4{}^{20}$$
$$(r_{21} - v_{23}) - (q_1{}^{43}p_4{}^{14} + q_4{}^{43}r_4{}^{14} + q_4{}^{14})(v_{14} -$$
$$v_{12}) - - q_4{}^{43}(v_{43} - v_{41}) + W_L = 0.$$

$$\delta a_2 - \delta \lambda_2 \sin \varphi_2 - \delta \lambda_1 + \delta \lambda_1 \sin \varphi_1 + v_{21} - v_{12} + W_a = 0.$$

If the latitude and longitude of apex No. 3 of the polygon (Figure 2), being computed from apex No. 1 clockwise, is denoted as B_{123}, L_{123} andfrom apex No. 1 counterclockwise as B_{143}, L_{143}, then the free terms $W_B = B_{123} - B_{143}$ and $W_L = L_{123} - L_{143}$ will conform to the equations (11) and (12).

The free term of equation (13) is found from the expression where a_{2n}, a_{1m} and A_{2m}, A_{1m} are the astronomic and geodetic (computed) azimuths of the initial lines 2 n and 1 m.

The coefficients of the sought for corrections in equations (11) and (12) are computed from the value of certain derivatives for which Helmert introduced the following designations:

$$P_1{}^{JK} = \frac{\partial B_K}{\partial b_J}, \quad q_1{}^{JK} = \frac{\partial L_{JK}}{\partial B_J},$$
$$P_3{}^{JK} = \frac{\partial B_K}{\partial s_{JK}}, \quad q_3{}^{JK} = \frac{\partial L_{JK}}{\partial s_{JK}}, \quad r_3{}^{JK} \frac{\partial A_{KJ}}{\partial s_{JK}}$$
$$P_4{}^{JK} = \frac{\partial B_K}{\partial A_{JK}}, \quad q_4{}^{JK} = \frac{\partial L_{JK}}{\partial A_{JK}}, \quad r_4{}^{JK} = \frac{\partial A_{KJ}}{\partial A_{JK}}$$

The corrections δa, $\delta \lambda$ to astronomic azimuths and longitudes were made only in the overall adjustment of polygons. However, in his work (Ref. 11), Krasovskiy, in investigating the azimuthal equation of a section, correctly notes: "We would have been able to also introduce into the azimuthal equation the corrections of the astronomically determined longitudes and azimuths, but it is obvious that these corrections would have been unreliably determined." (Page 10, first par.) It should be added that since longitudes and azimuths are common to several sections then in the separate adjustmentof the latter, corrections to one and the same astronomic elements acquire quite some significance in general. The method of eliminating the multi-significance of corrections was proposed later by Krasovskiy, that is, in connection with

the adjustment of 87 polygons of first order triangulation in the USSR.

4. Proposals of Urmayev and Pranis-Pranevich on the Adjustment of an Astronomic-Geodetic Net

After the adjustment of the eight polygons of the Astro-geodetic of the USSR, the search for a better method of computing first order triangulation did not stop. Besides Krasovskiy's work, work in this field was divided between N. A. Urmayev, and I. Yu. Pranis-Pranevich whose proposals were tested in experimental computations.

Urmayev's method, like Krasovskiy's, is based on the fact that the method of least squares is practically accepted for the adjustment of certain interdependent functional interrelationships. The difference in Urmayev's method (Ref. 12) and Krasovskiy's method of 1931 lies primarily in the use of an indirect method of measurements for the adjustment of polygonal first order triangulation.

Secondly, two corrections to the astronomic azimuth and longitude are substituted for by one correction u = to the Laplace azimuth. In the case, Urmayev's method, instead of condition (9) uses the requirement

This requirement was applied in the adjustment of 87 polygons of the astrogeodetic net in 1942.

In order to adapt the method of indirect measurements to the adjustment of first order polygonal triangulation, Urmayev derived (ref 12) three kinds of error equations. He introduced the sought for corrections into the functions of the corrections dB, dL and ---- the geographic corrdinates B, L and the orientation angles z^o (the o signifies an approximated value).

The presentation of these error equations give us Figure 3 and the following expressions:

$$\delta s_{12} = - \frac{M_1}{\rho''} \cos A_{12} dB_1 - \frac{M_2}{\rho''} \cos A_{21} dB_2 -$$

$$- \frac{N_2}{\rho''} \cos B_2 \sin A_{21} dL_2 - \frac{N_1}{\rho''} \cos B_1 \sin A_{12} dL_1 +$$

$$\frac{M_2}{s_{12}} \sin \left(5^o_{12} - s_{12} \right)$$

$$U_{12} = -\mathcal{l}_{21} + \frac{N_1}{S_{12}} \sin A_{12}\, d B_1 + \frac{M_2}{S_{12}} \sin A_{21}\, d B_2 - $$

$$- \frac{N_2}{S_{12}} \cos B_2 \cos A_{21}\, d L_2 - \frac{N_1}{S_{12}} \cos B_1 + \frac{M_2}{S_{12}} \sin$$

$$\left(\cos A_{12} - \frac{S_{12}}{N_1} tg\, B_1\right) d L_1 + + [A^0{}_{12} - \beta_{12}+z^0{}_1)] .$$

$$v_{21} = - \mathcal{l}_{21} + \frac{M_1}{S_{12}} \sin A_{12}\, d B_1 + \frac{M_2}{S_{12}} \sin$$

$$A_{12}\, d B_2 - - \frac{N_1}{S_{12}} \cos B_1 \cos A_{12}\, d L_1 -$$

$$\frac{N_2}{S_{12}} \cos B_2 \left(\cos A_{21} - - \frac{S_{12}}{N_2} tg\, B_2\right) d L_2 +$$

$$[A^0{}_{21} - (\beta_{12} + z^0{}_0)]$$

$$u_1 = \mathcal{l}_{21} - d L_1 \sin B_1 - L(\alpha_1 - z^0_1) + (\lambda_1 - L^0_1) \sin B_1 .$$

From these equations we see that for each apex of a polygon (except for the datum point) from the adjustment we must determine three corrections: dB, dL and $\mathcal{l}$. Urmayev con sidered the method of indirect observations, when used in the adjustment of an astro-geodetic, more simple and flexible in the organizational (overall) sense.

Let us now consider the proposals of Pranis-Pranevich, as expressed in the unpublished "Notes on methods and order of adjustment of first order triangulation", proposed to GUGK in 1939-40.

The "Notes" is composed of two sections. In the first are given the characteristics of methods of adjusting first order triangulation (simplified and rigid) while in the second is given an account of the method and program of adjustment computations.

Pranis-Pranevich did not support adjustment by dependent functions. He did not recognize the most widely accepted methods of Krasovskiy and Urmayev, believing that "With the aim of practically overcoming the problems of adjusting large measurements (dimensions of first order triangulation, the basic principle of least squares has been violated. In both methods the problem is not solved under the condition of minimum sum of squares of the corrections to the weight of independent (one from another) functions of measurement results. All functions of measurement results, the corrections to which are sought

under the condition of minimum, with the exception of functions from the astronomic elements $A - \lambda \sin \phi$, by the methods of Krasovskiy-Urmayev, are interrelated. In adjusting nets of polygons (step II) the forward and back directions of geodetic lines are values which are related (dependent) by pairs. They are functions of the measured directions of one and the same section. After that, the lengths of geodetic lines and their direction, since they are functions of the measured directions of one and the same section, are also related values in pairs. Finally, the selection of junction figures as a rigid (correct) base for scale and for positions on the earth ellipsoid for individual sections is illegitimate (untenable)."

Further: "A detailed study of the accuracy of first order triangulation, adjusted by the methods investigated here, shows that the greatest source of error in triangulation is not the indirect measurement (determination) of directions and not the bases, but the artificial creation in the second stage of geodetic coordinates of the unction points" (pp 2-3 "Notes).

It is difficult to find complete grounds for objecting to these critical notes of Pranis-Pranevich is we consider the latest achievements in the field of computing technology.

Instead of the method of Krasovskiy-Urmayev, Pranis-Pranevich proposed two variants of rigid adjustment of first order triangulation. A requirement common to both variants is that:

$$[\rho v^2] + [\rho_u u^2] = \min.$$

where u and P_u are the corrections and weight of the functions of astronomic elements which are independent of horizontal directions (Laplace azimuths $A = a - (\lambda - L) \sin \phi$) and v,p are the corrections and weight of the measured horizontal directions. Besides this, into the foundation of both variants is included "The theory of conditional equations between the most probable results of direct measurements and the most probable magnitudes of the non-measured determined values" (p 15 "Notes").

Let us explain this. Assume we have a system of q conditional equations with n corrections in the observed values and with f non-measured unknowns X

$$a_1 v_1 + a_2 v_2 + \ldots + A_1 x_1 + A_2 x_2 + \ldots + w_1 = 0$$

$$b_1 v_1 + b_2 v_2 + \ldots + B_1 x_1 + B_2 x_2 + \ldots + w_2 = 0$$

$$q_1 v_1 + q_2 v_2 + \ldots + Q_1 x_1 + Q_2 x_2 + \ldots + w_q = 0,$$

Setting up the requirement (vv) = min, we arrive by well known means to a system of (q + f) normal equations:

$$[aa]k_1 + [ab]k_2 + \ldots + A_1 x_1 + A_2 x_2 + \ldots + w_1' = 0$$
$$[ba]k_1 + [bb]k_2 + \ldots + B_1 x_1 + B_2 x_2 + \ldots + w_2 = 0$$
$$[qa]k_1 + [qb]k_2 + \ldots + Q_1 x_1 + Q_2 x_2 + \ldots + w_q = 0$$

$$A_1 k_1 + B_1 k_2 + \ldots + Q_1 k_q = 0$$
$$A_2 k_1 + B_2 k_2 + \ldots + Q_2 k_q = 0$$

$$A_1 k_1 + B_F k_2 + \ldots + Q_F k_q = 0$$

From the systems of equations (21a) we can form the expression of correlates k_i through the unknowns x_1, x_2......and the coefficients with them A_1, A_2 .... These expressions, substituted in (21b) bring about a system of normal equations with the unknowns x_1, x_2 ...., the solution of which allows us to find the terms $A_i X_i$, $B_i X_i$ ...., $Q_i X_i$ which enter equation (21a). Then it is possible to solve those equations and from the determined correlates k_i compute the corrections $v_i = a_i k_i \ b_i 2_k \ldots$.

In his first variant, Pranis-Pranevich proposed to subdivide a net of closed polygons into several parts, for instance, into three parts as shown in Figure 4. The boundaries of these parts are arcs of triangulation A_1 abcA_2 and N_1pqrN_2 . The equations of values of directions in triangulation are determined by means of geodetic coordinates (latitude and Longitude) and geodetic azimuths on the boundary points A_1, a, b, c, A_2, N_1, p, q, r, N_2. These coordinates and azimuths, or more precisely, the corrections to their approximated values, are the "non-measured unknowns".

(Figure 4)

In the above-mentioned "Notes", Pranis-Pranevich states that "conditional equations are formed separately for each part, into these equations enter, besides the most probable values of directions and astronomic elements, the most probable geodetic coordinates and geodetic azimuths of the initial lines of base nets. The latter are located on the boundaries between adjoining parts, also on external borders turned to (facing) the side not yet covered with geodetic nets."

The systems of normal equations which arise in each part are solved until such time as the correlations are eliminated. It follows that for each part there remain the elimination equations and the transformed normal equations with the non-measured unknowns. Then,

grouping together corresponding parts of the transformed normal equations, we obtain a general system of normal equations, that is, we use the well known method of Pranis-Pranevich for solving normal equations by subdividing the triangulation into parts (Ref 15).

The second variant differs from the first only in the organization of the computing process. In the second variant, the normal equations are set up for each section. They contain, in the form of unknowns, not only the correlates, but also the non-measured unknowns; the corrections to the geodetic coordinates and azimuths at the ends of the section. As a matter of fact, in this variant each section may be treated as a part of the triangulation for which we find the elimination equations of the correlates and transformed equations containing only the non-measured unknowns. There is no need here to go into greater detail for the second variant. This can be found in the works (Ref 13) or (Ref 14).

After subjecting the methods of Krasovskiy and Urmayev for the adjustment of astro-geodetic nets to serious criticism and going on record for rigid adjustment, Pranis-Pranevich nevertheless violated his own recommended rigidity in the two variants. He proposed that, before the simultaneous adjustment of the entire net, at the intersects of arcs, junction figures be separated and adjusted as free systems.

The angles obtained as a result of this adjustment were to be used as rigid (true). In this fashion there was created a source of decreased accuracy in the scale and orientation of the external sides of the junction figure since the lengths and azimuths of the initial sides of the base net were carried forward to these sides) the external sides). However, even in Krasovskiy's method, the separate adjustment of junction figures formed by intersecting arcs, also led to the lowering of the accuracy of the results of the final section adjustment.

5. Later Proposals of F. N. Krasovskiy on the use of the Method of Projecting and Adjusting Sections

In the simultaneous adjustment of first order triangulation in the European USSR by Krasovskiy's method, eight polygons laid onto the Bessel ellipsoid without any change whatsoever in the following: 1) the relationship of the lengths of the bases to the geoid and 2) the horizontal angles measured on the physical surface of the earth. These lengths and angles, together with the elements of Bessel's ellipsoid, served for the computation of geodetic latitudes, longitudes and azimuths. Such an order of determining geodetic coordinates without preliminary projecting of base lines and horizontal angles on the surface of a reference ellipsoid is known as the method of development (unrolling).

Krasovskiy's investigations in the derivation of the dimensions and shape of the earth showed that the dimensions of the Bessel ellipsoid were too small and did not suit the territory of the USSR. They also showed that the development of triangulation on the surface of an ellipsoid which was ill suited by dimensions and shape for the actual figure of the earth within the limits of the USSR, would inject into elements of triangulation, fundamental distortions of a systematic character. If, for instance, on the surface of an ellipsoid whose dimensions are too small, we develop (unroll) arcs of triangulation along a parallel, then it will be found to be turned systematically toward the equator.

We have here a distortion similar in character to the distortion caused by the action of the overall refractional field in arcs of triangulation.

Latitudinal displacements of geometric nature (source) distort the free terms of azimuthal equations and in long arcs reach significant proportions (ref 26).

Assume that the bending of an intermediate side 0-1 of an arc of triangulation of equilateral triangles relative to its true direction 0-n will be w; then each succeeding side, as shown in Figure 5, will bend relative to direction 0-n by 2 w, 3w, ..., nw. It can easily be seen that the latitudinal displacement of the last (end) point of the arc will be

$$\triangle X_n = \triangle x_1 + \triangle x_2 + \ldots + \triangle x_n = \frac{\omega''}{\rho''} s(1 + 2 + \ldots + n) = \frac{\omega''}{\rho''} s \frac{n(n+1)}{2}$$

For the angle of the overall bend to the south we have

$$\vartheta = \omega \frac{n+1}{2} = \frac{\triangle W_a}{2} + \frac{\omega}{2},$$

the error in the azimuthal free term is

$$\triangle W_a = \omega n$$

Using differential formulas of the second series, we find (Ref 26) for an equilateral section (section of equilateral triangles with sides of 25 km) along a parallel B = 52° that $0/w = dA_{JK} = + 0.''14$. We also obtain from (22), the latitudinal displacement X_n = .63 m while the distortion of the free term of the azimuthal equation will be $w_a = 8 = 1''.1$ when the section length is 200 km (n = 8).

The values X_n and W_a are far from being negligible (are not insignificant), therefore, these errors of geometric origin must not distort or obscure the errors of the measured elements of triangulation.

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In accordance with the method of projection developed by Krasovskiy, before the adjustment, triangulation bases are projected upon the reference ellipsoid while corrections for astrogeodetic deflection of the vertical are made to the measured horizontal directions. After this, the triangulation is adjusted. This allows the elimination of errors of geometric character in the astro-geodetic net regardless of the suitability of the adopted reference ellipsoid for the given territory. This is a very important feature, the singnificance of which is very difficult to overestimate.

The method of projection was first applied in the USSR in the simultaneous of 87 polygons on Krasovskiy's reference ellipsoid. In this way the astro-geodetic net of the USSR was freed of distortions of geometric origin.

It is noted above, that in the application of Krasovskiy's method in 1932 for the adjustment of eight first order polygons, corrections were not determined for Laplace azimuths in the section adjustments. Is is understood, that is was impossible to obtain a uniform system of such corrections from individual section adjustment. With the aim of improving the accuracy of azimuth determinations, Krasovskiy proposed (Ref 10) to determine every 800-1000 km using a special program, astronomic, azimuths and longitudes of higher accuracy. These so-called fundamental Laplace azimuths would be accepted as true values. Between fundamental azimuths there would be set up azimuthal equations which would include for each section, not only the correction $\sum \Delta JK$ for the horizontal direction of the section JK, but also the corrections u_J and u_K to the ordinary Laplace azimuths. Simultaneous solution of these equations by the method of least squares made possible the attainment of a uniform system of corrections to the ordinary Laplace azimuths – but not the fundamental.

The order of adjustment of sections in this case was as follows: 1) preliminary adjustment of sections for the conditions of figure, pole and base, 2) the setting up (from directions which have been preliminarily adjusted) and solving of systems of azimuthal equations of the following type

$$\sum \Delta_1 - u_1 + w_1 = 0$$

$$\sum \Delta_2 + u_1 - u_2 + w_2 = 0$$

$$\sum \Delta_{n-1} + u_{n-2} - u_{n-1} + w_{n-1} = 0$$

$$\sum \Delta_n + u_{n-1} + w_n = 0$$

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(corrections u_0 and u_n to the fundamental azimuths equal zero), 3)
correcting the ordinary azimuths with the corrections u_1, u_2 ....,
u_{n-1} and solving in each section the azimuthal equation including the
corrected free term $-\mathcal{L}\Delta u$ simultaneously with the equations for figure,
pole and base. Since the section had undergone a preliminary adjust-
ment for the stated three types of conditions, then in the second ad-
justment of the section, their fre e terms equalled zero.

In the transition from conditional equations (25) to the normal
it was recommended that a consideration be given to weights for $\angle \Lambda$
and u also. This problem is very difficult is we reject its formal
solution which is rarely satisfactory.

From the above it can be seen that the later studies of Kras-
ovskiy in the field of adjusting an astro-geodetic net led to his de-
tailed development of the method of projection which protects the net
from errors (distortions) of geometric character. Besides this, in
striving for an increase in the accuracy of azimuthal determinations,
Krasovskiy proposed including within the net, a system of concrete
fundamental azimuths. However, this proposal, which was not put into
practice, allowed the formation in this case of two different systems
of azimuthal corrections: 1) from the simultaneous solution of azi-
muthal equations by sections and 2) as the result of solving polygonal
equations in the simultaneous adjustment of systems of polygons.

6. Krasovskiy's Method and the Proposal of D. A. Larin in the Adjust-
ment of 1942.

For the proper application of Krasovskiy's method in the adjust-
ment of 87 polygons on the reference ellipsoid bearing his name, there
were compiled suitable instructions. (Ref 6). In the first part of
this instruction there is a detailed statement as to the order of pro-
jecting the astro-geodetic net of the USSR on Krasovskiy's reference
ellipsoid. Here are given, with proper explanations, formulas to cor-
rect horizontal directions for relative deflection of the vertical and
for elevation of the observed object (station). A formula is also
given for the reduction of measured directions in normal cross section
to geodetic lines.

Then follow instructions for computing corrections for the ef-
fects of various types of errors and applying them to the base line
measurements. Within these corrections there is apportioned a correc-
tion for projecting the measured length of a base on the surface of
the reference ellipsoid.

In this same part of the instructions, there is examined the or-
der of processing astronomic determinations (expressions are given for
corrections in order to change to the system of catalog FKZ, for osi-
cillation of the pole, and others.)

The second part is especially interesting. In this section, adjustment by Krasovskiy's method with minor deviations, is given in detail step by step for practical use.

In view of the absence of fundamental azimuths in the astro-geodetic net, the instructions reflect another of Krasovskiy's proposals on adjusting sections, i.e., the above described azimuthal equations (5) are set up for all sections after their preliminary adjustment and are solved simultaneously. It follows that though the first proposal of Krasovskiy excluded determination of corrections to fundamental azimuths the second stipulates the determination of corrections to all Laplace azimuths except the initial (Pulkovo).

It is also seen from the contents of the instructions that the adjustment of polygons is carried out under the condition

$$\left[p_s \delta s^2 \right] + \left[p_v\, v^2 \right] + \left[p_v\, v^2 \right] = min.$$

that is, corrections a and λ to the astronomic azimuths and longitudes (contrary to Krasovskiy's proposals) are substituted by a single correction $u = \lambda - \lambda sin \phi$ to the Laplace azimuths. Here is also given the practical order of determing the weights p_s, p_v and P_u.

The schemes for conducting the computations as proposed by D.A. Larin for various stages of the adjustment, give the entire adjustment continuity and harmony. Especially essential is the proposal of Larin for setting up conditional equations of coordinates for systems of polygons. Each polygon is independently projected on a plane in the Gauss Kruger projection in which case a local origin of coordinates is selected for the first point of each polygon while the plane coordinates of other vartices are found from geodetic coordinates. Thus, in the application of Krasovskiy's method for the adjustment of 87 polygons, complicated setting up of coordinate equations into geographic coordinates was done away with. In accomplishing this portion of adjustment computations D. A. Larin then used a second simpler and novel method of compiling polygonal equations on a plane. This method is not reflected in the instructions (Ref 6) but it is explained in our work (Ref 16). The essence of this method lies in the use of Legendre's theorem for substituting plane angles for spherical in polygons and later setting up conditional corrdinate equations on a plane. We shall explain this briefly.

Assume (Figure 6) a pentagonal polygon of the astro-geodetic net (among the 87 polygons there were some with more vertices). We subdivide it for our study into three triangles. One of the variants of such a subdivision gives us spherical triangles JKG, KGE, and KFE with corresponding angles n = B_{JG} - P_{JKs} X 1, W_1; X_{II}, W_{II}, t_{II} and X_{III}, ϕ t_{III}

Figure 6

Having computed the spherical excess of these triangles by the known formula (Ref 10).

$$\varepsilon = \varepsilon' \left(1 + \frac{m^2 n}{8} \right),$$

where, for instance, for triangle JKG we have

$$\varepsilon_{JKG} = \rho'' \frac{s_{JK} \, s_{JG} \sin g}{2 R_m^2}$$

$$n = \frac{1}{3} \left(n_J + n_K + n_G \right)$$

$$n_J = \frac{1}{R_J^2}, \quad n_K = \frac{1}{R_K^2}$$

$$n_G = \frac{1}{R_G^2}$$

$$m^2 = \frac{1}{3} \left(s_{JK}^2 + s_{JG}^2 + s_{KG}^2 \right)$$

The side entering here s_{KG} can be obtained from the expression

$$s_{KG}^2 = s_{JK}^2 + s_{JG}^2 - 2 s_{JK} \, s_{JG} \cos g.$$

Further, we obtain on a plane, with correctional terms for spheroidity

$$n' = n - \frac{1}{3} \varepsilon_{JKG} - \frac{1}{60} \varepsilon_{JKG} \, n \left(m^2 - s_{KG}^2 \right) - \frac{1}{12} \varepsilon_{JKG} \left(\frac{n_J}{n} - 1 \right)$$

Having thus determined the other angles also on planes
and ω' we can compute the directional angles χ', ψ', τ χ'_I, ω'_I,

$$\chi_{KF} = A_{JK} + 180^\circ - \chi'$$

$$\chi_{FE} = \chi'_{FK} - \psi' \text{ u. т. п.}$$

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The conditional equations of coordinates on a plane will be (ref 16)

$$\sum_{i=2}^{i=n} \cos \alpha_{i-1,i}\; \delta s_{i-1,i} + \sum_{i=2}^{i=n} \frac{y_n - y_i}{\rho''}\left(v_{i,i-1} - v_{i,i+1}\right) + W_x = 0$$

$$\sum_{i=2}^{i=n} \sin \alpha_{i-1,i}\; \delta s_{i-1,i} - \sum_{i=2}^{i=n} \frac{x_n - x_i}{\rho''}\left(v_{i,i-1} - v_{i,i+1}\right) + W_y = 0$$

$$W_x = \sum s \cos \alpha, \quad W_y = \sum s \sin \alpha$$

Here, in contrast to the coordinate equations introduced in the instructions (ref 6), the coefficients with corrections δs and v do not contain scale coefficient m_1 which occurs in the use of Gauss' projection. The azimuthal equations were united with the corrdinate euqations

$$v'_K - v'_J + v_{KJ} - v_{JK} + N_{JK} = 0$$

$$N_{JK} = (A'_{Jm} - \alpha_{Jm}) - L_J - \lambda_J \sin \varphi_J - [(A_{KN} - \alpha_{KN}) - (L_K - \lambda_K) \sin \varphi_K]$$

which were set up for each side JK of the polygons. In equation (35), A',a' indicate the corresponding computed geodetic and astronomic azimuths of the initial sides Jm, Kn of the section.

In the astro-geodetic net of the USSR which was adjusted in this order there were 310 sections of triangulation and one traverse forming 87 polygons. Therefore, in the sectional adjustment, there had to be simultaneously solved 311 azimuthal equations of type (5) while in the adjustment under the above named condition (15), there were solved simultaneously 174 coordinate equations (34) and 311 azimuthal equations (35).

Certainly, it could have been foreseen earlier that the corrections to the Laplace azimuths obtained from the solution of 311 equations of type (5) will be essentially different from the corrections of like type obtained from the solution of a system of 485 oequations For an evaluation of the character of the discrepancies between these corrections, Table 2 is included below with a few corrections obtained from the solution of various systems of equations.

TABLE II

Azimuths at the ends of sections:			Corrections u from solving 311 equa.		Corrections u' from solving 485 equa.	
J		K	u_J	u_K	u'_J	u'_K
Bezhetsk		Shimsk	+ 2".46	+ 0".70	+ 1".77	-0".15
Gomel'		Orsha				
Tambov		Anna				
Agyz		Kazan'				
Neya		Vologda				
Krasn. Kuduk		Biryuzak				
Zima		Irkutsk				
Birobidzhan		Khabarovsk				

Krasovskiy did not publish his proposition concerning the simultaneous solution of azimuthal equations (5); therefore, it was never made clear, what purpose had the author in allowing the derivation of two systems of azimuthal corrections.

7. B. N. Rabinovich's Investigation of the Order for Utilizing Laplace azimuths.

Elimination or reduction of systematic errors is of great importance in a large astro-geodetic net, where its deformation acquires systematic character and leads to the distortion of orientation and of triangulation scale. The main sources of systematic deformation of the net are: (1) developing the triangulation on the surface of a reference ellipsoid which has dimensions unsuitable for the given territory; (2) effect of overall refraction field caused by heat transfer from equator to pole. If the development of triangulation is done on a reference ellipsoid of undersized dimensions the, as already noted, the resulting deformation will be similar to that which is caused by the effect of general refraction field. However, in the astro-geodetic network of the USSR the effect of source one is excluded, because of the application of projection method. Bending of that net toward equator, due to the effect of source two, is established, according to B. N. Rabinovich's investigation (Ref. 4), by free terms of azimuthal and base equations, set up for long triangulations arcs from Pulkovo to the east (arcs along meridians).

Accumulation of systematic refractional distortions in these free terms is characterized by inequalities given above (2), (3), and (4). The best way to rectify the distortions is to use skillfully Laplace azimuths in making adjustment of the astro-geodetic net.

Investigating the effect of lateral refraction, the German geodetists Forster and Schutz (Ref. 22) have discovered that in the arc of the first-order triangulation along 52° parallel between Greenwich and Warsaw (some 1,600 km long) latitudinal shift of the Warsaw station toward the equator is somewhat more diminished, if corrections to Laplace azimuths are not sought in making the general adjustment of the triangulation by Helmert's method. Namely in this case, the shift will be diminished by 52.66 m, while in seeking corrections to Laplace azimuths it will be diminished only by 45.11 m. However, even after the publication of the work (Ref. 22) in 1929, the geodetists of the USSR and of other countries continued to follow the order established by Helmert in using astronomic elements in triangulation as values which receive corrections from its adjustment.

It is difficult to indicate the reason, why neither in the Soviet Union, nor abroad, was the order of using the Laplace azimuths revised after the publication of work (Ref. 22). Possibly, the high reputation enjoyed by Helmert was here decisive. Or, it may be, that the experiment of Forster and Schutz did not appear convincing, since the large

shifts could be attributed to distortions of geometric origin, due to plotting the Greenwich-Warsaw arc on Bessel's reference ellipsoid of undersized dimensions. Actually, corresponding computations show (Ref 26) that for this arc the latitudinal shift of geometric origin is $\triangle X_{\mu}$ = 36.4 m. Thus, over two thirds of the 53-m shift indicated in the work (Ref 22) can be attributed to the method of development (unrolling).

The results of B. N. Rabinovich's investigation which is based on the very extensive data of the astro-geodetic net of the USSR, processed by projection method, are free from distortions of geometric origin and therefore are reliable. In analyzing the deformation effect in the triangulation of the Cis-Caucasus anomaly field of latitudinal refraction (occurring on the northern cold slope of the ridge) it was established (Ref 4) that the arc Armavir - Bak (some 900 km long) appears bent to the north in case of purely geodetic adjustment, while it is less bent, if the Laplace azimuths are taken as correct. This situation is illustrated by data on the Table 3, where φ' and φ indicate the latitudes of junctions points, obtained from two adjustments, of which one was with the determination of corrections to Laplace azimuths, and the other without it.

TABLE 3

Junction points	$(\varphi' - \varphi)''$	$(\varphi' - \varphi)^{\mu}$
Armavir	0",0000	0,000
Georgiyevsk	-0 ,0011	-0,034
Gudermes	-0 ,0023	-0,071
Makhach-Kala	-0 ,0012	-0,037
Derbent	-0 ,0029	-0,090
Baku	-0 ,0161	-0,497 m

The data of this Table makes it doubtful, whether the order established by Helmert for the utilization of Laplace azimuths is correct. To verify this, an analysis was made of true residual errors obtainable from the elements of a model which was distorted and then adjusted by one and the other method for utilizing Laplace azimuths.

The model of polygons used for this purpose consisted of 48 sections formed by 18 quadrangular polygons with sides of 180, 210 and 232 km. Angles in the section were distorted by systematic errors due to the effect of a homogenous refraction field, according to the formula

$$\delta_{pg} = \delta_o \frac{S_{pg}}{30} \ sin \ \sigma_{pg}$$

where σ_{pg} is azimuth of the side, S_{pg} is its length, and ____ is the refraction for the side s_o, situated normally to the gradient of air density.

The distorted model was adjusted twice: with Laplace azimuths ajusted (variant I), and rigid (variant II). Thus, in overall adjustment of the model by variant I, the above mentioned equations (34) and (35) were developed and simultaneous solved. In all, there were 84 equations, of which 36 were of the first type, and 48 of the second type, according to the number of sections. In applying the variant II, we have in the equations (35) $u'_K = u'_J = 0$, and $W_{JK} = 0$; therefore we arrive at the equality $v_{KJ} = v_{JK}$. This equality is equivalent with $\triangle A_{JK} = v_{JK}$, but with a weight $P_A = 2 \ P_V$, and the equations (34) and (35) in variant II are substituted (Ref 18) by the following system of coordinate equations:

$$\sum_{i=2}^{i=1} cos A_{i-1,i} \Delta s_{i-1,i} - \sum_{i=2}^{i=N} \frac{y_i - y_{i-1}}{P_{ii}} \Delta A_{i-1,i} + W'_x = 0$$

$$\sum_{i=2}^{i=N} sin A_{i-1,i} \Delta s_{i-1,i} + \sum_{i=2}^{i=N} \frac{x_i - x_{i-1}}{P_{ii}} \Delta A_{i-1,i} + W'_y = 0$$

$$W'_x = \sum s \ cos A, \qquad W'_y = \sum s \ sin A.$$

Here $A_{i} - 1,i$ are azimuths of the diagonals of sections, while ____ are corrections to the lengths and to azimuths of these diagonals.

In solving the system of equations (34), (35) and the system of equations (36) by the method of least squares, the following conditions are assumed:

(a) for the first variant

$$[P_S \delta_S ^2] + [P_V r^2] + [P_u u^2] = min$$

(b) for the second variant

$$[P_S \Delta_S ^2] + [P_A \Delta A^2] = min$$

From the comparison of deviations of the adjusted values of coordinates from their true values x_o, y_o, for both variants, it was established (Ref 17) that deviations

$$\delta x_i = x_i - x_o$$

$$\delta y_i = y_i - y_o$$

corresponding to the variant I of equation, considerably exceed the deviations

$$\delta x_{ii} = x_{ii} - x_o$$

$$\delta y_{ii} = y_{ii} - y_o$$

obtained in adjusting by variant II. Residual systematic lateral shifts
are in case of variant I almost 10 times larger than the shifts after
adjustment by variant II (Ref 9).

Therefore, Helmerts approach to the utilization of Laplace azi-
muths, as adjusted values, is less acceptable for the reduction of
systematic lateral shifts, than variant II of adjustment.

However, there arises the question: is not variant I leading to
more accurate results in case of adjustment of a net distorted only by
random errors? In analyzing this question, it was taken into account
that both variants of adjustment are not strictly rigorous, since the
minimum conditions (37a) and (37b) are imposed on the sum of squares of
interdependent functions.

Moreover, in comparing both variants, the following consideration
must be kept in mind: the practical application for the adjustment of
astro-geodetic network of a non-rigorous method which requires consid-
erable amount of labor can be justified only under the condition that
this method produces always more accurate results, than the other non-
rigorous method which is less labor-consuming.

The accoracy of adjustment results was determined by the value
of the mean absolute error remaining in the corrdinates of the ends of
sections or vertexes of polygons, after adjustment of the distorted
models. Naturally, when the trinagulation, distorted by a certain ag-
gregate $()$ of random errors, is adjusted by two variants, the degree of
compensation of the effect of those errors in functional dependencies,
(for example, in the above mentioned corrdinates) depends upon the dis-
tribution of the aggregate $()$ of errors over the adjusted elements,
and upon the applied variant of adjustment.

In B. N. Rabinovich's work (Ref 19) it is shown that in an arc,
consisting of n sections, adjusted by two variants, there exists for
each section JK the following relationship between the true residual
errors of azimuth of A_{JK} of the diagonal

$$\delta A_{JK} = \Delta A_{JK} + (u_J + v_{JK} + \delta(u_J))$$

where A_{JK} and ΔA_{JK} are residual errors of azimuths corresponding to
variants I and II. $\delta_J = A_{Jm} - A_{Jm}$ is the difference of azimuths (La-
place azimuth and geodetically computed one) for the initial side Jm;
u_J and v_{JK} are corrections to Laplace azimuth $A_{(Jm)}$ and angle β_{JK}
respectively. In the same work it was established that, when n is
sufficiently large, we have in the arc

$$\sum^{n} (\delta A)^2 > \sum^{n} (\Delta A)^2$$

Since the values entering into this inequality are basic for the
determination of the mean square root error $\sqrt{\sum \Delta^2}$, then the
accuracy of the azimuths of diagonals in variant I of the adjustment is
lower than the accuracy obtained in variant II. In other words, the
lateral shifts of the ends of diagonals, remaining after the adjustment,

must be larger in variant I, than the same shifts in variant II. Since, however, the compensation of errors in the corrdinates depends upon the distribution of the given aggregate of random errors, then, even with the existing inequality (39), the mean absolute residual error of coordinates in variant II may be either larger or smaller, than the residual error of corrdinates, resulting in variant I. These assertions were confirmed by the results of experimental adjustment (Ref 19). Experimental adjustment of polygon models by two variants has shown (ref 9) that the residual shift of vertex most remote from the initial point may be in variant I either be larger or smaller, than the shift of the same vertex, remaining after adjustment in variant II. Thus, out of two non-rigorous variants of adjusting the astrogeodetic net for random errors, the more labor-consuming variant I does not always assure a more accurate result. Moreover, variant I always reduces less the effect of systematic errors due to the action of the general field of lateral refraction. All this leads to the conclusion that there is no sufficient basis for using the Laplace azimuths as concrete values for the subsequent adjustment of astro-geodetic network of the USSR.

8. New propositions Concerning the Adjustment of the Astro-geodetic Net of the USSR

Experience with inserting lower-order nets within the first-order triangulation framework adjusted in 1932, has demonstrated that the effect of errors of this framework sometimes essentially distorts the measureddirections in the area triangulation. The same happens in case of adjustment of modern second-order area nets based on the adjusted elements of 87 polygons of astro-geodetic network, projected on Krasovskiy's reference ellipsoid. The cause of this phenomenon basically are the error in the adjusted elements of the said network. In order to reduce the effect of these errors, it is necessary to coordinate the accuracy and construction principles of the 1-st order and 2-nd order triangulations, and to observe as strictly as possible the conditions of the application of least squares method to the adjustment of the astro-geodetic network.

It should be remembered that the accuracy of angle measurement in the 1-st order and 2-nd order triangulation in the USSR is typified by the mean square errors of $\pm 0".7$ and $\pm 1".0$ respectively. These values differ but little from each other, and thus the effect of the first error will not be negligible in comparison with the effect of the second error.

Non-observance of the criterion of negligibility for the errors of astro-geodetic net, in relation to the errors of area net of tne 2-nd order triangulationfollows also from the character of distribution of extra data in these networks: the adjusted elements of the second netowork are derived on the basis of considerably more extra data, than those in the first network. Extra data in the adjusted part of the 1-st

order triangulation of the USSR (87 polygons) make up about 80%, while in the area net of the 2-lnd order triangulation they exceed 200% (Ref 20).

Thus, the first-order triangulation, by its structure itself, cannot guarantee more accurate determination of the position of points on the ground than the area triangulation of second order. Because of this circumstance, it becomes necessary to raise somewhat the weight of the elements of polygonal astro-geodetic net by increasing it in the amount of extra data, after adding to it the most carefully triangulated parts of the second-order area triangulation. There have been made certain propositions concerning this question.

The author of the present article, writing on the problem which was discussed in TsNIIGA i K (Central Scientific Research Institute of Geodesy, Aerial Surveying and Cartography) in 1951, pointed out the possibility of increasing considerably the extra data in the polygonal astro-geodetic net, by supplementing it with diagonals, determined according to the diagonal arcs of triangles, taken from second-order area triangulation net. Utilization of these diagonals a, b, c, (Figure 7) doubles the amount of extra data arising in the system of quadrangles. Actually, in order to determine gk-1 vertices, the length and the azimuth of gk-1 diagonals must be known, i.e. the number of the necessary values is $p = 2(gk-1)$ (where g and k indicate the number of arcs along meridians and parallels respectively). Figure 7

The number of additional lengths and azimuths of diagonals will be $2(g-1)(k-1)$ in case of quadrangular polygons, while in adding the diagonals a, b, c... just as much extra data will be added, and their number $Q = 4(g-1)(k-1)$. Consequently, the ratio of the additional data to the number of the necessary ones will be

$$\frac{Q}{P} = \frac{2(g-1)(k-1)}{gk-1}$$

When $g = 41$ (distance about 9000 km) and $k = 12$ (about 2400 km) we obtain for 440 polygons $Q = 1.80$, or 180% of extra data, i.e. 100% more than without diagonals P a, b, c.. (Ref 20).

The percentage of the extra data is increased even more, if two intersecting diagonals p and q (Figure 7) are introduced into each polygon from data of second-order area triangulation net.

However, this proposition ignores the conditions of strict application of least squares method, since the dependent functions are taken, as the values to be adjusted. But, after all the great achievements of our time in the development of electronic computing machines for the solution of linear equations, it would hardly be justifiable to abbreviate the system of equations in the net just to make the computation work easier, by replacing the directly measured values with their interdependent functions, thus violating the rigor of adjustment and, consequently, decreasing the accuracy of results.

Another proposition for the construction and adjustment of the state basic geodetic network was made by Professor A. I. Durnev (Ref. 25). His suggestion to provide the main triangulation with bases and Laplace azimuths, in order to increase the amount of additional data, deserves attention, as conducive to the increase in weight of the elements of astro-geodetic network.

However, the principles of adjustment, offered in this writing (Ref 25), are contradictory to the conditions of rigorous adjustment; they resemble the proposals made by Hungarian geodesists (Hazay, Tarczy-Hornoch) regarding the substtitution of large "fictitious" triangles (side length 150--200 km) for the block of small triangles.

As a matter of fact, the angles and the length of sides of large triangles are computed from the elements of the small triangles, and thus are their interdependent functions. Tarczy-Hornoch points to the mutual dependence of angles and lengths of sides of fictitious triangles and, in order to weaken this dependence, he proposes to measure in the center of each block of triangles its own base line (Ref 23). However, this does not restore the rigor of adjustment; each consecutive placing of a block of small triangles in the corresponding adjusted large triangle introduces greater errors into the elements of the net being placed, than those errors which would appear in the same elements in case of a simultaneous adjustment of the net, without dividing into blocks of small triangles.

It must be noted that there is in the proposals regarding the adjustment of functiional dependencies a certain point which deserves attention and practical application: This is the possibility of increasing the amount of additional data in the astro-geodetic net by including into it the arcs or blocks of triangles of second-order area triangulation. However, the main thesis of these proposals, chvisaging the adjustment of computed values of dependent functions, could be justified only in that case, if it were practically impossible to perform the difficult computations of rigorous adjustment. At present, when fast-operating electronic computers are being rapidly developed, the efforts of Soviet geodesists in the field of adjustmentof astro-geodetic net of the USSR should not be directed toward a simplified, non-rigorous solution of this problem. It would be unjustified to concentrate geodetic thought on the application of the method of least squares to the adjustment of interdependent functions, the more so, that the practical work on adjustment will begin not earlier, than after 7-8 years, when experience in use of electronic computers for the solution of very large systems of linear equations will make this task considerably easier.

There is, however, a more rigorous method for the adjustment of the astro-geodetic net, which was worked out in detail by a group of geodsists consisting of N. V. Avayev, Ye. Ye. Biryukova, I. N. Landis, A. I. Petrova, and L. V. Zinevich (Ref 21).

The basic features of this method are: (1) adjustment by the method of a conditional observations with non-measured unknowns, and (2) division of the net into sections according to the multigroup solution of the system of equations by the method of Pranis-Pranevich.

Thus, this method is based on the propositions of Pranis-Pranevich, made in connection with the impending adjustment of 87 polygons of the astro-geodetic net of the USSR.

However, Pranis-Pranevich, as already noted, has violated the rigor of the method by excluding junction figures from simultaneous adjustment of the entire net. In Pranis-Pranevich's proposition, the corrections to the approximated values of geodetic coordinates of the junction points and geodetic azimuths of the initial sides are non-measured unknowns.

The proposition of the group of geodesists envisages a simultaneous solution of all equations of the astro-geodetic network (including also equations in the junction figures) under the condition $(pv^2) = $ min. Corrections v are determined to the measured angles only. It is correctly assumed that the Laplace azimuths and bases do not receive corrections from the adjustment. In order to shorten the time necessary for adjustment, the multigroup method of Pranis-Pranevich is applied, dividing the net into portions, the boundaries of which can be any sides of the sections in the triangulation. Those sides which serve as the boundaries of portions, are called by authors "the dividing lines" (parezy). As non-measured unknowns serve the corrections $\delta_i, \delta s_i, \delta x_i, \delta y_i$, according to the approximated values of the directional angles of the dividing sides, the length of these sides, and coordinates of one of their ends (adjustment is done on a plane).

Thus, in addition to the conditional equations of figures, horizon and poles, the following conditions will arise for each portion of the astro-geodetic net, bounded by k dividing lines:

(1) Directional angles $\quad \sum_1^n a_i v_i + \sum_1^k A_p \delta_{\sigma_p} + W_a = 0$

(2) bases $\quad \sum_1^{iv} b_i v_i + \sum_1^k B_p \delta s_p + W_s = 0$

(3) abscisses $\quad \sum_1^r c_i v_i + \sum_1^k A'_p \delta x_p + \sum_1^k B'_p \delta s_p + \sum_1^k C_p \delta x_p + W_x = 0$

(4) ordinates $\quad \sum_1^n d_i v_i + \sum_1^k A''_p \delta_{\sigma_p} + \sum_1^k B''_p \delta s_p + \sum_1^k D_p \delta y_p + W_y = 0$

Among these equations, we have, according to the terminology of Pranis-Pranevich, partially independent and partially connecting equations.

Although in the method of dividing lines the adjustment is done by angles, and not by independently measured directions, nevertheless the simultaneous solution of all equations, formed in the astro-geodetic net, is accomplished by this method completely. This is a great step toward the increase of rigor in the adjustment of the polygonal first-order triangulation of the USSR.

The method of dividing lines would become absolutely rigorous, if not the angles, but the pertinent directions were adjusted, since the method of angle measurement with the same weight in all combinations, as accepted in the USSR, at each station leads to a number of independently measured directions. This fact was taken into account by D. A. Larin in his 1958 draft of instructions for the adjustment of the astro-geodetic net of the USSR. The method of adjustment, as set up in these instructions, is based on the following: (1) projection of the net on the surface of reference ellipsoid, (2) preserving unchangeable the lengths and Laplace aizimuths of initial sides, and (3) simultaneous solution of all equations of the net under the condition $(pv^2) = min$, where v are corrections to the directions, independently measured at each station. Solution of equations is done by multigroup method of Pranis-Pranevich. Naturally, the procedure and the practical details of a rigorous adjustment of the enormous astro-geodetic net of the USSR STIll have to be worked out thoughtfully and carefully. It must be taken into account that the adjustment of directions in the net increases the number of computations; since we have to do here with a very large net, it is necessary to ascertain on the basis of preliminary investigation, how far is it possible to accomplish the enormous volume of computation work even with the help of available improved devices and methods.

In order to bring the values of weights p nearer to the reality in the physical sense, they should be established according to mean square errors, derived from non closures of triangles, and not from the number of operations; then the regional values of weights thus obtained, evidently will have to be assigned to large massifs of the astro-geodetic net.

If g and k represent the number of triangles forming the astro-geodetic net along the direction of meridians and parallels respectively, then we have in such a net quadrangular polygons and sections

$$P = (g-1)(k-1)$$

$$r = 2g + (g+k) - 2$$

Considering that in some regions of the USSR it is admissible, according to the decision of GUGK (Main Administration of Geodesy and Cartography) Committee, to increase the area of polygons from 40,000 sq km to 60,000 sq km, (Ref 24). It may be accepted for orientation g = 41 and k = 12; then p = 440 and r = 930. Since in a section of the first-order triangulation there are on the average 15 triangles (as per data from 87 polygons), then the number of conditional equations of figures will be about 14,000. The number of azimuthal and base equations will be 2 X 930 = 1,860, and the number of coordinate conditions 2 X 440 = 880. Added to it should be the conditional equations of poles, which arise in some sections and in junction figures. Therefore, when the astro-geodetic net of the USSR is completed, the total number of conditional equations will be of the order of 20,000.

Naturally, in this system of equations the relationship between the equations of considerably remote partys of the net will be weak. Essentially, the system of 20,000 equations will be regarded as consisting of 4-5 "independent" or barely dependent systems, and there is no special reason to fear the difficulties of solving them and uniting them into a single connected system. Even at present, the Moscow aerogeodetic Establishment of GUGK has positive experience in solving simultaneously a system of 3,800 equations. Of course, it is not absolutely necessary to adhere to the method of conditional observations. Possibly, the method of indirect observations may prove to be more convenient whatever the method, the total connecting system of equations will hardly include more than 6,000 equations. Nevertheless, there will be an enormous volume of computing work which has to be carefully planned in advance. Of great importance for the solution of this problem are the resolutions adopted by the "Committee for working out the measures connected with the construction and adjustment of the state geodetic net of the USSR". This Committee which was organized by the GUGK of the Ministry of Interior of the USSR conducted its work in summer 1957 and, concerning the adjustment of the astro-geddotic net of the USSR, has adopted resolutions No. 2 and No.6 (Ref 24). The first of these resolutions specifies fundamental propositions for the impending adjustment of the astro-geodetic net, and the second points out a number of investigations of which particularly important is the improvement of methods and techniques in solving the system of algebraic linear equations. Namely, the practical experience in the use of the method for accurate solution of very large systems of linear equations by means of electronic computers will make it possible to perform a completely rigorous adjustment of the astro-geodetic net of the USSR, and, consequently, to attain a greater accuracy of the adjusted elements.

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